Logic For Computer Science Exam 29/01/2020 B

1. Define logical equivalence for propositional formulas. How can logical equivalence be established by using resolution or related methods?

2. How can the boolean function described by a formula be computed? What is the boolean function described by:

$$(P \Leftrightarrow (\neg Q)) \wedge (P \vee (Q \Rightarrow S)) \ ?$$

3. What is a clause? What is a resolvent of two clauses? Let C_1, C_2, C be clauses such that C is a resolvent of C_1, C_2. What is the relation between the formulae corresponding to clauses C_1, C_2, C? Compute a resolvent of:

   (a) $\{\neg P, Q, R\}$ and $\{P, Q, S\}$,

   (b) $\{P, Q\}$ and $\{P, \neg R\}$,

   (c) $\{P, \neg Q, \neg R\}$ and $\{P, Q, R\}$.

4. What is a complete set of logical connectives? Show that $\{\bot, \Rightarrow\}$ is a complete set of logical connectives.

5. Let $A[x]$ be a predicate logic formula where x is a free variable, and G one such that x is not free in it. Prove

$$\forall x\ (A[x] \Rightarrow G) \Leftrightarrow (\exists x\ A[x] \Rightarrow G).$$

6. Decide, using a resolution-based method, whether the clause set:

   (1) $\{\neg P, Q, \neg R\}$,

   (2) $\{P, R, S\}$,

   (3) $\{\neg P, \neg Q, S\}$,

   (4) $\{P, \neg Q, \neg R\}$,

   (5) $\{\neg P, Q, \neg S\}$,

   is satisfiable or not. If yes, give a satisfying truth valuation.

7. Define the value under interpretation and variable assignment for expressions in predicate logic. Consider the following language, containing:

   - Function symbols $\mathcal{F}$: $+$ binary, $-$ unary, $*$ binary.
   - predicate symbols $\mathcal{P}$: $=, <, \leq$ all binary.
   - constant symbols $\mathcal{C}$: $0, 1$.

   Choose an interpretation of this language in the universe of sets. For this interpretation, choose a variable assignment, and compute the value under the chosen interpretation and variable assignment for the following expressions:

   - $(x + (-y)) * z$,
   - $(x * y + (-z)) \leq (-z + 1) * 0$.

8. Describe the one literal rule, used by the Davis Putnam and Davis Putnam Logemann Loveland methods. Show that if the formula corresponding to the clause set obtained after the application of this rule is satisfyable, then the formula corresponding to the clause set before the application of the rule is also satisfiable.